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Efficiency of the variable fidelity design optimization method with variable complexity models

Eficiencia del Método de Optimización del Diseño de Fidelidad Variable con Modelos de Complejidad Variable

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Scaling methods, Different degree of nonlinearity, Different number of design variables, Variable fidelity

ABSTRACT

The goal of this investigation is to provide a deeper understanding about a variable fidelity optimization algorithm and some scaling methods through three test problems. The first two problems are analytic, and the third one is a structural optimization problem. The test problems have been specifically constructed to look for insights regarding the use in the algorithm of models (high fidelity and low fidelity models) with different degree of nonlinearity and different number of design variables. Performance of the variable fidelity framework for first order and second order scaling methods (multiplicative and additive), is compared to a standard sequential quadratic programming optimization performed on the high fidelity model. The main contributions of this investigation are the insights gained with the specifically constructed test problems, which can be extended to other problems, about the use of a trust region variable fidelity framework, and the choice of the most suitable scaling methods depending on the case study at hand. In addition, results show how a reduction in the design cycle time can be obtained, by reducing the number of high fidelity function calls while achieving convergence.

PALABRAS CLAVE:

Métodos de escalamiento, Diferente grado de no-linealidad, Diferente número de variables de diseño, Fidelidad variable

RESUMEN

El objetivo de esta investigación es mejorar la comprensión del algoritmo de fidelidad variable y de diversos métodos de escalamiento a través de tres problemas. Los primeros dos problemas son analíticos, y el tercero es un problema de optimización structural. Los problemas han sido construidos especificamente para comprender el funcionamiento del algoritmo con modelos (alta y baja fidelidad) de diferente grado de no-linealidad y diferente número de variables de diseño. El rendimiento del algorimo al usar diversos métodos de escalamiento de primero y segundo orden (aditivo y multiplicativo), es comparado con el rendimiento de usar programación cuadrática secuencial solamente sobre el modelo de alta fidelidad. La principal contribución de esta investigación es la comprensión ganada con los problemas propuestos, lo cual puede extenderse a otros problemas, sobre el alcance y limitandes del algoritmo, y la elección del método de escalamiento más apropiado dependiendo del caso de studio que se tenga. Además, los resultados muestran como una reducción del tiempo de diseño puede obtenerse, mientras se reduce el número de evaluaciones al modelo de alta fidelidad y se alcanza convergencia.

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1 INTRODUCTION

In the design process the low fidelity models are typically much cheaper to evaluate, but designs produced using these models neglect important physics or details included in the more expensive higher fidelity models. Variable fidelity model management methods have been developed to solve optimization problems that involve computationally intensive simulation. Linking the optimizer to the low fidelity model makes the design process tractable. The high fidelity model is sampled after each sequence of low fidelity optimization as part of a trust region model management strategy that drives convergence of the optimization to the high fidelity solution [1-5]. A formal proof of convergence was developed in [2,3], that ensures that the optimization process converges to a solution of the original problem. In the variable fidelity algorithm, scaling methods produce a transformation or scaling function that can transform the low fidelity model to match the high fidelity model.

In the literature one can find methods that work with a similar motivation as variable fidelity methods (conduct optimization on an inexpensive surrogate model). Some of these methods are: successive approximate optimization (SAO) algorithms [6], space mapping techniques [7], and others that sequentially refine the surrogate model through the optimization process [8,9]. Variable fidelity methods can further reduce computational cost by using a function similar to a response surface, where a low fidelity model provides a global approximation which is locally corrected using a scaling function to produce a better overall approximation of the high fidelity function.

In the literature one can find significant work that make use of variable fidelity optimization, and the most relevant to our investigation are described next.

A first-order model management optimization methodology was presented in [10] for solving high-fidelity optimization problems with minimal expense in high-fidelity function and derivative evaluation. The applicability to general models was demonstrated on two computational studies of aerodynamic optimization, where the differences between the high and low fidelity models was on the mesh size and in the equation that is used (Navier-Stokes or Euler equations). Variable-complexity optimization was used in [11], to aerodynamic shape design problems with the objective of reducing the total computational cost of the optimization process. The differences between the high and low fidelity models considered the use of different levels of fidelity in the analysis models, and the use of different sets of design variables. Results showed that variable-fidelity methods using different physical models were not as successful as those which used the same model. The authors argued that no correction could compensate for the omission of the physical characteristics of the problem, and that the method was only relevant for optimization problems for which all the design variables had a similar physical meaning. In [12], it was shown that first-order consistency can be insufficient to achieve acceptable convergence rates in practice, and presented new second-order additive, multiplicative, and combined corrections which can significantly accelerate convergence of surrogate-based optimization (SBO) methods. The authors presented three test problems, and the main goal of the investigation was to show the superior converge rate of second order corrections in all the test problems.

It is important to note that in the aforementioned works [10,11], both the high and low fidelity models used the same number of variables. On the other hand, even though the work in [12] compares different scaling models in examples where the high and low fidelity models have variable complexity, the studies are somehow general. This work goes deeper on the characteristics and circumstances under which the additive scaling method can be preferred over the multiplicative, and vice versa, and a first order method can be preferred over the second order method.

Therefore, the goal of this investigation is to show new insights about a trust region variable fidelity algorithm and scaling methods through three specifically constructed test problems. Performance of the variable fidelity framework for first order and second order scaling methods (multiplicative and additive), is compared to the standard sequential quadratic programming (SQP) optimization performed on the high fidelity model. The insights gained can be extended to other problems, about the use of the framework with models that differ in the degree of non-linearity and in the number of design variables, to choose the most suitable scaling methods depending on the case study at hand.

2 METHODOLOGY

This section presents the variable fidelity model management framework, and a brief description of the first and second order multiplicative and additive scaling methods is presented.

2.1 VARIABLE FIDELITY (VF) IN DESIGN

In the design process a common engineering practice is to drive the preliminary design using lower fidelity models as surrogates of expensive high fidelity simulations. Higher fidelity models are then used in the final design stages to refine the design. However, using automated optimization methods at this stage may still require enormous computational resources. The variable fidelity model management framework addresses this problem by incorporating both models, the lower fidelity model and the higher fidelity model, into one optimization framework [13]. The low fidelity models are scaled to approximate the simulations results based on high fidelity models. Therefore, optimization can be performed using mainly low fidelity simulations, reducing the overall computational cost, while requiring only a few high fidelity simulations to update a scaling function that drives the low fidelity simulations towards the optimal design.

2.2 THE VARIABLE FIDELITY OPTIMIZATION ALGORITHM

The framework used in this investigation for variable fidelity optimization is depicted in Figure 1 [13], and is designed to reduce the number of high fidelity function calls during the optimization process by using a scaling function and lower fidelity models. The following steps are the basic steps of the framework:

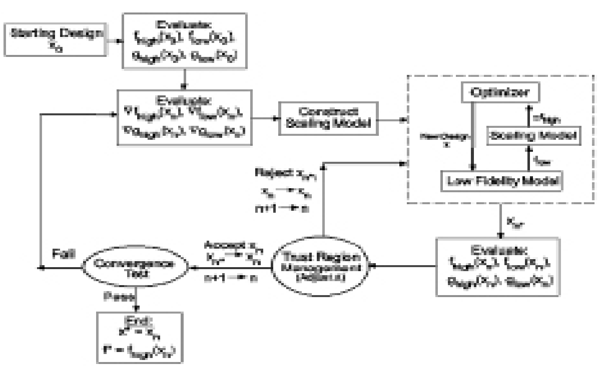

Figure 1. Variable fidelity model management framework.

Step 1 (Initialization): At the starting design point x0, the objective function is evaluated using both fhigh(x0) and flow(x0), the high and low fidelity models respectively. All of the high and low fidelity inequality and equality constraints ghigh(x0), glow(x0), hhigh(x0) and hlow(x0) are also evaluated. In addition the initial trust region size is specified.

Step 2 (Gradient evaluation): The gradients of the objective for both the high and low-fidelity models are evaluated at the current design point xn, as well as the Jacobian for the constraints.

Step 3 (Construct scaling model): A scaling model is constructed to ensure matching between the different fidelity models. This model can be based on many different methods; additive and multiplicative are the most common, see section 2.3.

Step 4 (Optimize scaled low fidelity model): The optimization process is carried out using the SQP optimizer from the function fmincon, which is provided in the Matlab's Optimization Toolbox.

Step 5 (Evaluate new design and F penalty function): A new candidate point xn* is found as a result of solving the optimization problem in step 4. At this new candidate point, the high-fidelity objective, fhigh(xn*), and constraints ghigh(xn*) and hhigh(xn*) are evaluated. The objective and constraint values are used to calculate a current value of the penalty function F for the high-fidelity and scaled low-fidelity models, which will be used in step 6. The penalty function is defined as

$$F(x)=f(x)+1/\mu\_n \sum \max(0,g_i (x))+ 1/\mu\_n \sum|h j (x)| \quad (1)$$

In Equation (1), μ is the penalty weight that is typically decreased by a factor of 10 each time a new point is accepted. This penalty weighting drives all the active constraints to zero as the algorithm converges.

Step 6 (Trust region management): In order to help guarantee convergence of the variable fidelity optimization framework, a trust region model management strategy is employed. This method provides a means for adaptively managing the allowable move limits for the approximate design space. A trust region ratio allows the trust region model management framework to monitor how well the approximation matches the high fidelity design space. The trust region ratio, ρn, is calculated at the new candidate point xn*, as

$$\rho_n=(F(xn)high-F(x\hat{}*n)high)/(F(xn)scaled-F(x\hat{}*n)scaled \quad (2)$$

In Equation (2), F()high and F()scaled are the penalty functions for the high and scaled low fidelity models, and the point xn is the current best design at iteration n. The trust region size is governed by the following standard rules [14]:

$$\Delta(n+1)=\begin{cases} c1\Delta n:\rho n\leq R1 \lor \rho n\geq R\_3 \\ \Delta n:R1<\rho n>R2 \\ \Gamma\Delta n:R2<\rho n>R3\ ) \end{cases}$$

In Equation (3), $\Gamma=c2$ if $\Vert x^{\wedge *}n\text{-}xn\ \Vert\infty=\Delta n$, otherwise $\Gamma=1$. A typical set of values for the range limiting constants are R1=0.25, R2=0.75 and R3=1.25, while the trust region multiplication factors are typically c1=0.3 and c2=2. If ρ is near one, the approximation is quite good. If ρ is near zero, then the approximation is not as good, but it still captures the minimization trend. If ρ is negative, then the point is a worse design. In this case the point is rejected, the trust region size is reduced by the factor c1, and the algorithm returns to step 4. As long as ρ>0, the point is accepted and the algorithm proceeds to step 7.

Step 7 (Convergence Test): Convergence can be determined by the following stopping criterion:

$$\Vert\Vert fhigh\ (xn\ )\text{-}fhigh\ (xn\text{-}1)\ \Vert\Vert / \Vert\Vert fhigh\ (xn\text{-}1)\ \Vert\Vert <\varepsilon\_f \quad (4)$$

$$\Vert\Vert xn\text{-}xn\text{-}1)\Vert\Vert\ /\ \Vert\Vert\ xn\text{-}1\ \Vert\Vert <\varepsilon\_x \quad (5)$$

In Equations (4) and (5), εf and εx are tolerances supplied by the user, and n is the current iteration counter. If any of the two inequalities in Equations (4)-(5) at the current point are true, the algorithm is considered converged. If the convergence test is true, then the final design is found, otherwise, the algorithm returns to step

2..3 SCALING METHODS

The additive and multiplicative scaling models are used to construct an unknown scaling function to update the lower fidelity model, which in turn, will approximate the higher fidelity model. Each method can be modeled as first order or second order, and the goal is to match the gradient and curvature of the high fidelity model respectively. It is important to remember that at least first order matching between the scaled low fidelity model and the high fidelity model is required for proof of convergence.

2.3.1 MULTIPLICATIVE SCALING MODEL

This method was devised by Alexandrov and Lewis [15] based on [16]. For the first order multiplicative scaling method see [17], and for multiplicative second order scaling see [18,19]. As described in step 3 from the variable fidelity optimization algorithm, a given set of high and low fidelity models, fhigh(x) and flow(xr), where (xr x), can be matched by multiplying the low fidelity model by an unknown function β(x), which is called the multiplicative scaling function. This is posed mathematically as

$$fhigh\ (x)=\beta(x)\ flow\ (x^{\wedge}r) \quad (6)$$

Solving for the unknown multiplicative scaling function results in

$$\beta(x)=fhigh\ (x)\ /flow\ (x^{\wedge}r) \quad (7)$$

From inspection of Equation (7), it is shown that the function β(x) is the scaling ratio of the high fidelity model to the low fidelity model, and when it is multiplied by the low fidelity model, the high fidelity model is achieved. However, the exact scaling function β(x) is not known and must be approximated.

2.3.2 FIRST ORDER AND SECOND ORDER MULTIPLICATIVE SCALING.

At a given design point, for example the current design, the scaling function is defined as

$$\beta(xn\ )=fhigh\ (xn\ )/flow\ (xn^{\wedge}r) \quad (8)$$

This scaling factor at any other point can be approximated using a Taylor series to first order

$$\tilde{\beta}(x)\approx\beta(xn\ )+\nabla\ \beta(xn\ )^{\wedge}T\ \ (x\text{-}xn) \quad (9)$$

To evaluate this, the gradient information is needed and can be obtained by differentiating Equation (8), resulting in

$$\nabla\beta(x_n)=\begin{bmatrix} \dfrac{f_{low}(x_n^r)\left.\frac{\partial f_{high}}{\partial x_1}\right|_{x=x_n}-f_{high}(x_n)\left.\frac{\partial f_{low}}{\partial x_1}\right|_{x=x_n}}{f_{low}(x_n^r)^2} \\ \vdots \\ \dfrac{f_{low}(x_n^r)\left.\frac{\partial f_{high}}{\partial x_m}\right|_{x=x_n}-f_{high}(x_n)\left.\frac{\partial f_{low}}{\partial x_m}\right|_{x=x_n}}{f_{low}(x_n^r)^2} \end{bmatrix} \quad (10)$$

where m is the number of design variables.Therefore, a first order update on the low fidelity model is:

$$f_{high} \approx \tilde{\beta}(x)\, f_{low} \quad (11)$$

This model ensures that at the initial design point, the updated low fidelity model matches the function and the gradient of the high fidelity model. The identical process is done in order to scale each constraint.

An analogous approach to the first order method is followed to obtain the multiplicative second order scaling method. The scaling factor can be approximated using a Taylor series to second order

$$\tilde{\beta}(x) \approx \beta(x_n) + \nabla\beta(x_n)^T \Delta x + 1/2\, \Delta x^T \nabla^2 \beta(x_n) \Delta x \quad (12)$$

To evaluate Equation (12), the Hessian information is needed and can be obtained by differentiating Equation (10). Computing this symmetric full rank matrix would be quite expensive; therefore, Hessian update methods such as BFGS and SR1 can be used to compute these terms. Now a second order update on the low fidelity model can be obtained, and it has the same form as in the first order method (Equation (11)).

2.3.3 ADDITIVE SCALING MODEL

The additive method was presented by Lewis and Nash [20]. For additive second order scaling see [18,19].

A given set of high and low fidelity models, fhigh(x) and flow(x^r), where (x^r ⊂ x), can also be matched by adding the low fidelity model to an unknown function γ(x). This is expressed mathematically as.

$$f_{high}(x) = f_{low}(x^r) + \gamma(x) \quad (13)$$

In Equation (13), solving for the additive scaling function results in

$$\gamma(x) = f_{high}(x) - f_{low}(x^r) \quad (14)$$

It is clear that when this function is added to the low fidelity model, the response of the high fidelity model is obtained.

2.3.4 FIRST ORDER AND SECOND ORDER ADDITIVE SCALING

In a similar fashion to the first order multiplicative scaling method, the additive scaling function at a given design point has the value

$$\gamma(x_n) = f_{high}(x_n) - f_{low}(x_n^r) \quad (15)$$

This additive scaling factor at any other point can be approximated using a Taylor series to first order

$$\tilde{\gamma}(x) \approx \gamma(x_n) + \nabla\gamma(x_n)^T (x - x_n) \quad (16)$$

which requires gradient information that can be obtained by differentiating Equation (15). This gives

$$\nabla\gamma(x_n) = \nabla f_{high}(x_n) - \nabla f_{low}(x_n^r) \quad (17)$$

Therefore, a first order update on the low fidelity model is

$$f_{high} \approx f_{low} + \tilde{\gamma}(x) \quad (18)$$

This model ensures that at the current design point, the updated low fidelity model matches both the function and the gradient of the high fidelity model exactly, which is required for proof of convergence.

An analogous approach to the first order method is followed to obtain the additive second order scaling method. The scaling factor can be approximated using a Taylor series to second order

$$\tilde{\gamma}(x) \approx \gamma(x_n) + \nabla\gamma(x_n)^T \Delta x + 1/2\, \Delta x^T \nabla^2 \gamma(x_n) \Delta x \quad (19)$$

To evaluate this, the Hessian information is needed and can be obtained by differentiating Equation (17). Computing this symmetric full rank matrix would be quite expensive, that is why Hessian update methods such as BFGS and SR1 are used to compute these terms. Now a second order update on the low fidelity model can be obtained, and it has the same form as in the first order method (Equation (18)).

3 TEST CASES

This section will present the three problems that have been specifically constructed to provide a deeper understanding about a variable fidelity optimization algorithm and some scaling methods. In addition, the three test problems are used to compare the savings offered by each scaling method and their convergence.

Two analytic problems were created to study the influence of having lower number of variables in the low fidelity model, and of being of lower order, with respect to the high fidelity model, where one problem uses a two-dimensional high fidelity model of degree 6, and the other uses a four-dimensional high fidelity model of degree 4. A third problem, a high-performance low-cost structure, was created to study the influence of having significantly lower number of variables in the low fidelity model, with respect to the high fidelity model.

In order to solve the problems, first order and second order scaling for the multiplicative and additive methods were implemented in the variable fidelity framework. Traditionally the cost of the second order methods has been prohibitive. In order to completely avoid the extra function calls required to compute the symmetric full rank Hessian matrix, consecutive first order information is used to approximate it. Two of the most popular methods to approximate the Hessian are the BFGS and SR1 methods. In all test problems, all gradient information was obtained using forward finite differences.

In this investigation, an important question that is intended to be answered is whether or not the results and behavior of the scaling methods, in terms of convergence and required high fidelity functions calls, would be similar in all test problems while keeping constant certain characteristics (difference in number of variables and degree of nonlinearity between the HF and LF models). In addition, the three proposed test problems are intended to help in analyzing a variety of case study scenarios, in order to be able to decide what type of scaling method is preferred, depending on the case study at hand and the difference between the high and low fidelity models.

In order to compare the methods, the number of high fidelity function calls was computed for each method. For comparison purposes, the number of function calls needed for a standard SQP optimization performed on the high fidelity model alone is also presented for each problem. In addition, the results will show how a reduction in the design cycle time can be obtained by using a cheap and simple low fidelity model, while reducing the number of high fidelity function calls and achieving convergence to the high fidelity optimal design.

Analytic Problem 1

This problem is similar to the two dimensional model presented in [13], and aims to study the influence of having lower number of variables in the low fidelity model, and of being of lower order, with respect to the high fidelity model.

The high fidelity model is a degree four high fidelity model with four design variables. On the other hand, the most complex low fidelity model is a variation of the high fidelity model, which adds linear and nonlinear noise factors to change the shape of the design space and location of the optima.

Table 1 shows the high fidelity (HF) model, and fourteen low fidelity (LF) models with variable complexity, where the most complex is four dimensional of degree four, and the less complex is a constant. Note that constraints are not shown for all the low fidelity models, however constraints are considered equal among models with equal number of design variables.

HF Model (4 variables, degree 4)	$f_{high} = 4x_1^2 + x_2^3 + 2x_3^2 + 5x_4^3 + x_1x_2^2 + x_1x_3 + x_1x_4^2 + x_2^2x_3 + x_2^2x_4^2 + x_3x_4^2$ $g_{high} = \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4} - 2 \le 0$
LF Model (4 variables, degree 4)	$f_{low} = 4(x_1+0.1)^2 + (x_2-0.1)^3 + 2(x_3+0.1)^2 + 5(x_4-0.1)^3 + x_1x_2^2 + x_1x_3 + x_1x_4^2 + x_2^2x_3 + x_2^2x_4^2 + x_3x_4^2 + 0.1$ $g_{low} = \frac{1}{x_1} + \frac{1}{x_2+0.1} + \frac{1}{x_3} + \frac{1}{x_4+0.1} - 2 - 0.1 \le 0$
LF Model (4 variables, degree 3)	$f_{low} = 4(x_1+0.1)^2 + (x_2-0.1)^3 + 2(x_3+0.1)^2 + 5(x_4-0.1)^3 + x_1x_2^2 + x_1x_3 + x_1x_4^2 + x_2^2x_3 + x_3x_4^2 + 0.1$
LF Model (4 variables, degree 2)	$f_{low} = 4(x_1+0.1)^2 + (x_2-0.1)^2 + 2(x_3+0.1)^2 + 5(x_4-0.1)^2 + x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4 + 0.1$
LF Model (4 variables, degree 1)	$f_{low} = 4(x_1+0.1) + (x_2-0.1) + 2(x_3+0.1) + 5(x_4-0.1) + 0.1$
LF Model (3 variables, degree 3)	$f_{low} = 4(x_1+0.1)^2 + (x_2-0.1)^3 + 2(x_3+0.1)^2 + x_1x_2^2 + x_1x_3 + x_2^2x_3 + 0.1$ $g_{low} = \frac{1}{x_1} + \frac{1}{x_2+0.1} + \frac{1}{x_3} - 2 - 0.1 \le 0$
LF Model (3 variables, degree 2)	$f_{low} = 4(x_1+0.1)^2 + (x_2-0.1)^2 + 2(x_3+0.1)^2 + x_1x_2 + x_1x_3 + x_2x_3 + 0.1$
LF Model (3 variables, degree 1)	$f_{low} = 4(x_1+0.1) + (x_2-0.1) + 2(x_3+0.1) + 0.1$
LF Model (2 variables, degree 3)	$f_{low} = 4(x_1+0.1)^2 + (x_2-0.1)^3 + x_1x_2^2 + 0.1$ $g_{low} = \frac{1}{x_1} + \frac{1}{x_2+0.1} - 2 - 0.1 \le 0$
LF Model (2 variables, degree 2)	$f_{low} = 4(x_1+0.1)^2 + (x_2-0.1)^2 + x_1x_2 + 0.1$
LF Model (2 variables, degree 1)	$f_{low} = 4(x_1+0.1) + (x_2-0.1) + 0.1$
LF Model (1 variable, degree 3)	$f_{low} = 4(x_1+0.1)^2 + 0.1$ $g_{low} = \frac{1}{x_1} - 2 - 0.1 \le 0$
LF Model (1 variable, degree 2)	$f_{low} = 4(x_1+0.1)^2 + 0.1$
LF Model (1 variable, degree 1)	$f_{low} = 4(x_1+0.1) + 0.1$
LF Model (0 variables, degree 0)	$f_{low} = 0.1$ $g_{low} = -2 - 0.1 \le 0$

Table 1. High fidelity model and low fidelity models for test problem 1.

The variable fidelity model management framework used separately the HF model with each of the 14 LF models, which resulted in 14 case studies. For all these cases, the initial design point considered was [5,5,5,5] T, and the design variables were bounded between 0.5 and 8. In addition, all the methods converged to the same solution, the optimum of the high fidelity model, and all used the same convergence criteria of εx = εf = 0.0001.

The use of the SQP optimizer on the high fidelity model resulted in the optimum design of [2.3, 1.9, 2.7, 1.5]T, with function value of 102.8, and a total of 13 iterations and 82 function evaluations were required for convergence.

Figure 2 shows the number of high fidelity function calls required by the variable fidelity model management framework for the 14 cases studies, as a result of using the aforementioned multiplicative (Mul) and additive (Add) scaling methods (first order, second order BFGS, and second order SR1). Also, for comparison purposes with the SQP results, the 82 high fidelity function evaluations required by the SQP optimizer are represented by a horizontal red line. In addition, this figure shows that the most competitive methods with respect to the required SQP HF function evaluations are: second order additive BFGS (Add BFGS), second order additive SR1 (Add SR1), and first order additive (Add 1st).

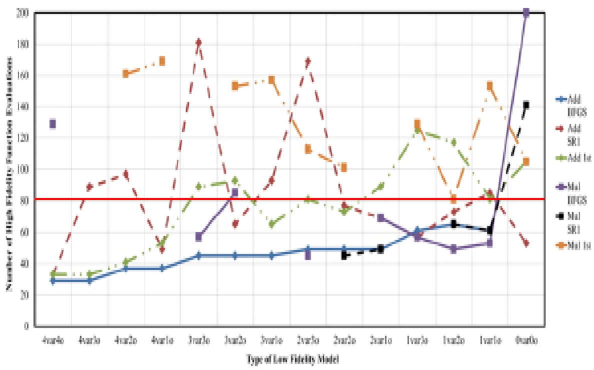

Figure 2. Variable Fidelity Optimization results for analytic test problem 1.

The results from Figure 2 can be summarized in Table 2, where the best scaling methods can be easily identified for case scenarios where there could be a small or big difference between the HF and LF models. In the last section, the results from Table 2 will be compared to the results obtained in the other test problems to get to a final conclusion.

Case scenarios	Best scaling methods		
Small difference in # of variables	Add. BFGS	Add. 1st	
Big difference in # of variables		Add. 1st	Add. SR1
Small difference in the degree of nonlinearity	Add. BFGS	Add. 1st	
Big difference in the degree of nonlinearity	Add. BFGS	Add. 1st	Add. SR1
Small total difference	Add. BFGS	Add. 1st	
Big total difference		Add. 1st	Add. SR1

Table 2. Best scaling methods for different case scenarios

3.2 ANALYTIC PROBLEM 2

The Barnes Problem [21] will be used as a high fidelity model to study the influence of having lower number of variables in the low fidelity model, and of being of lower order, with respect to the high fidelity model. The problem consists of a degree six high fidelity model with two design variables, and even though the problem has only two design variables, the high-order nonlinearities make it challenging to solve.

Table 3 shows the high fidelity (HF) model, and 13 low fidelity (LF) models with variable complexity, where the most complex is two dimensional of degree six, and the less complex is a constant. Note that constraints are not shown for all the low fidelity models, but constraints are equal among models with equal number of design variables.

The variable fidelity model management framework used separately the HF model with each of the 13 LF models, which resulted in 13 case studies. For all these cases the initial design point considered was [30,30]T, and the design variables were bounded between 0 and 70. In addition, all the methods converged to the same solution, the optimum of the high fidelity model, and all used the same convergence criteria of εx = εf = 0.0001.

The use of the SQP optimizer on the high fidelity model resulted in the optimum design of [49.5, 19.6] T, with function value of -31.6451, and a total of 9 iterations and 30 function evaluations were required for convergence.

HF Model (2 var., deg. 6)	$f_{high} = a_1 + a_2x_1 + a_3x_1^2 + a_4x_1^3 + a_5x_1^4 + a_6x_2 + a_7x_1x_2 + a_8x_1^2x_2 + a_9x_1^3x_2 + a_{10}x_1^4x_2 + a_{11}x_2^2 + a_{12}x_2^3 + a_{13}x_2^4 + \frac{a_{14}}{x_2+1} + a_{15}x_1^2x_2^2 + a_{16}x_1^3x_2^2 + a_{17}x_1^3x_2^3 + a_{18}x_1x_2^2 + a_{19}x_1x_2^3 + a_{20}e^{a_{21}x_1x_2}$ $g_{1high} = \frac{x_1x_2}{700} - 1 \geq 0$ $g_{2high} = \frac{x_2}{5} - \frac{x_1^2}{25^2} \geq 0$ $g_{3high} = \left(\frac{x_2}{50} - 1\right)^2 - \left(\frac{x_1}{500} - 0.11\right) \geq 0$
LF Model (2 var., deg. 6)	$f_{low} = a_1 + a_2x_1 + a_3x_1^2 + \lvert a_4\rvert x_1^3 + a_7x_1x_2 + a_8x_1^2x_2 + a_9x_1^3x_2 + a_{10}x_1^4x_2 + a_{11}x_2^2 + a_{12}x_2^3 + a_{13}x_2^4 + \frac{a_{14}}{x_2+1} + a_{15}x_1^2x_2^2 + a_{16}x_1^3x_2^2 + a_{17}x_1^3x_2^3 + a_{18}x_1x_2^2 + a_{19}x_1x_2^3 + a_{20}$ $g_{1low} = \frac{x_1x_2}{750} - 0.5 \geq 0$ $g_{2low} = \frac{x_2}{5} - \frac{x_1}{4^2} \geq 0$ $g_{3low} = \left(\frac{x_2}{50} - 1\right)^2 - \left(\frac{x_1}{500} - 0.11\right) \geq 0$
LF Model (2 var., deg. 5)	$f_{low} = a_1 + a_2x_1 + a_3x_1^2 + \lvert a_4\rvert x_1^3 + a_7x_1x_2 + a_8x_1^2x_2 + a_9x_1^3x_2 + a_{10}x_1^4x_2 + a_{11}x_2^2 + a_{12}x_2^3 + a_{13}x_2^4 + \frac{a_{14}}{x_2+1} + a_{15}x_1^2x_2^2 + a_{16}x_1^3x_2^2 + a_{18}x_1x_2^2 + a_{19}x_1x_2^3 + a_{20}$
LF Model (2 var., deg. 4)	$f_{low} = a_1 + a_2x_1 + a_3x_1^2 + \lvert a_4\rvert x_1^3 + a_7x_1x_2 + a_8x_1^2x_2 + a_9x_1^3x_2 + a_{11}x_2^2 + a_{12}x_2^3 + a_{13}x_2^4 + \frac{a_{14}}{x_2+1} + a_{15}x_1^2x_2^2 + a_{18}x_1x_2^2 + a_{19}x_1x_2^3 + a_{20}$
LF Model (2 var., deg. 3)	$f_{low} = a_1 + a_2x_1 + a_3x_1^2 + \lvert a_4\rvert x_1^3 + a_7x_1x_2 + a_8x_1^2x_2 + a_{11}x_2^2 + a_{12}x_2^3 + \frac{a_{14}}{x_2+1} + a_{18}x_1x_2^2 + a_{20}$
LF Model (2 var., deg. 2)	$f_{low} = a_1 + a_2x_1 + a_3x_1^2 + a_7x_1x_2 + a_{11}x_2^2 + \frac{a_{14}}{x_2+1} + a_{20}$
LF Model (2 var., deg. 1)	$f_{low} = a_1 + a_2x_1 + \frac{a_{14}}{x_2+1} + a_{20}$
LF Model (1 var. x₁, deg. 6)	$f_{low} = a_1 + a_2x_1 + a_3x_1^2 + \lvert a_4\rvert x_1^3 + a_5x_1^4 + a_{16}x_1^5 + a_{17}x_1^6 + a_{20}$ $g_{1low} = \frac{x_1}{750} - 0.5 \geq 0$ $g_{2low} = -\frac{x_1}{4^2} \geq 0$ $g_{3low} = -\left(\frac{x_1}{500} - 0.11\right) \geq 0$
LF Model (1 var. x₁, deg. 5)	$f_{low} = a_1 + a_2x_1 + a_3x_1^2 + \lvert a_4\rvert x_1^3 + a_5x_1^4 + a_{16}x_1^5 + a_{20}$
LF Model (1 var. x₁, deg. 4)	$f_{low} = a_1 + a_2x_1 + a_3x_1^2 + \lvert a_4\rvert x_1^3 + a_5x_1^4 + a_{20}$
LF Model (1 var. x₁, deg. 3)	$f_{low} = a_1 + a_2x_1 + a_3x_1^2 + \lvert a_4\rvert x_1^3 + a_{20}$
LF Model (1 var. x₁, deg. 2)	$f_{low} = a_1 + a_2x_1 + a_3x_1^2 + a_{20}$
LF Model (1 var. x₁, deg. 1)	$f_{low} = a_1 + a_2x_1 + a_{20}$
LF Model (0 var., deg. 0)	$f_{low} = a_1 + a_{20}$ $g_{1low} = -0.5 \geq 0$ $g_{2low} = -\frac{1}{4^2} \geq 0$ $g_{3low} = +0.11 \geq 0$

Table 3. High fidelity model and low fidelity models for test problem 2.

Figure 3 shows the number of high fidelity function calls required by the variable fidelity model management framework for the 13 case studies, as a result of using the aforementioned multiplicative (Mul) and additive (Add) scaling methods (first order, second order BFGS, and second order SR1). Also, for comparison purposes with the SQP results, the 30 high fidelity function evaluations required by the SQP optimizer are represented by a horizontal red line. In addition, this figure shows that the best method with respect to the required SQP HF function evaluations is the second order additive SR1 (Add SR1), but in general all the other methods are very competitive, depending on the type of LF model.

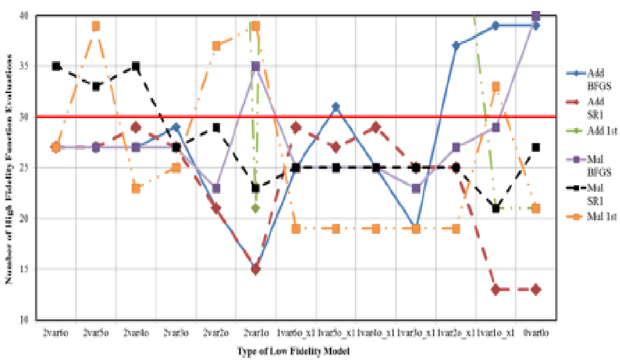

Figure 3. Variable Fidelity Optimization results for analytic test problem 2.

The results from Figure 3 can be summarized in Table 4, where the best scaling methods can be easily identified for case scenarios where there could be a small or big difference between the HF and LF models. In the last section, the result from Table 4 will be compared to the result obtained in the other test problems to get to a final conclusion.

Case scenarios	Best scaling methods		
Small difference in # of variables	Add. SR1	Add. BFGS	Mul. BFGS
Big difference in # of variables	Add. SR1		
Small difference in the degree of nonlinearity	Add. SR1	Add. BFGS	
Big difference in the degree of nonlinearity	Add. SR1	Add. 1st	
Small total difference	Add. SR1	Add. BFGS	Mul. BFGS
Big total difference	Add. SR1	Add. 1st	

Table 4. Best scaling methods for different case scenarios.

3.3 Structural Optimization Problem

This is a structural optimization problem in which the objective of the design is to minimize the weight of the structure while maximizing the payload capability. The multi-objective optimization is transformed into a single objective optimization via a cost-performance index. The problem aims to study the influence of having significantly lower number of variables in the low fidelity model, with respect to the high fidelity model. Figure 4 and Figure 5 show the physics-based high and low fidelity structures to be optimized respectively.

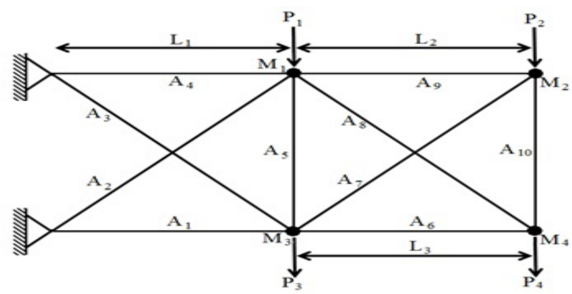

Figure 4. Physics-based high fidelity structure.

The high fidelity model was introduced in [22-23] and consists of a total of 17 design variables (cross sections, trusses topology and payloads) and 13 inequality constraints. The objective is to find the size and shape of the truss such that the weight (Wtot) of the structure is minimum (low cost) and the loads (Pi) it is capable of sustain and the payload (Mi) it carries are a maximum (high performance). This multi-objective problem is formulated as a single objective problem by defining a cost-performance index (CPI) which includes each of the objectives as shown in Equation (20). The design variables in this model are the length of the rectangular first bay (L1) and the top and bottom lengths of the outer bay (L2, L3), the masses (payload) placed on all the unconstrained nodes (M1-4), and the areas of truss members (A1-10).

The design is decomposed into three disciplines or contributing analysis. The configuration design calculates the weight of the structure (Wtot) and the loads (Pi) applied to the structure. The structural design group calculates the deflections (δi) and stresses (σi) of the truss members. This analysis is performed using a finite element code in which the truss members are modeled as rod elements capable of sustaining axial loads only. A dynamic response design subspace calculates the fundamental frequency (fn1) of the structure.

The structural material used is Aluminum, with the following properties: elastic modulus E= 70 GPa, yield strength σy=95 MPa, and density ρ=2770 kg.m-3.

The high fidelity problem is stated as follows [22,23]:

$$Minimize: f_{high}(x)=CPI=w1\,Wtot+(w2/(\sum Pi)+(w3/\sum Mi)$$

Subject to: $g1=1-(Mtot)min/(\sum Mi)\geq 0$

$g2=1-(Ptot)min/(\sum Pi)\geq 0$

$g\_3=1-f(n1,min)/fn1 \geq 0$

$g4\text{-}13=1-|\sigma 1\text{-}10|/\sigma y \geq 0$

$M^{(1)}(i)\leq Mi\leq Mi^{(u)}\quad i=1,..,4$

$L^{(l)}(k)\leq Lk\leq Lk^{(u)}(k)\quad k=1,..,3$

$A^{(l)}(m)\leq Am\leq Am^{(u)}(m)\quad m=1,..,10$

where w1=0.003, w2=106, w3=3.5x106, (Mtot)min=2600 kg, (Ptot)min=50,000 N, fn1,min=15 s-1, M(l)1-4= 100 kg, M(u)1-4=3,000 kg, L(l)1-3=2 m, L(u)1-3=30 m, A(l)1-10=5x10-4 m2, A(u)1-10=0.01 m2. The loads P1-4 are defined to be functions of the lengths of the bays L1-3 and the payload masses M1-4:

$$Pi=\sum^{(3)}(k)=1\,a^{(i)}(k)(Lk/Lref)^{(-b(k^{i}))}+\cdots\sum^{4}(j)=1c^{i}(j)\,(Mj/Mref)^{(d^{i}(j))} \quad (21)$$

with Lref= 10 m, Mref=650 kg. The coefficients a,b,c, and d were taken from [22,23].
The initial design point considered for all case studies was [10,10,10,800,800,800,800,0.002,0.002,0.002,0.002,0.002,0.002,0.002,0.002,0.002,0.002]T.

The use of the SQP optimizer on the high fidelity model resulted in the optimum design of [2,3.5,2,830.6,3000,3000,3000,0.0005,0.01,0.0005,0.0021,0.0005,0.0005,0.0005,0.0005, 0.0005, 0.0005]T, with function value of 358.3, and a total of 25 iterations and 453 function evaluations were required for convergence, see Table 5. The convergence criteria used for all case studies related to the structural optimization problem are εx = εf = 0.0001.

The physics-based low fidelity multi-objective problem is formulated as a single objective problem, which includes each of the objectives as shown in Equation (22). In order to relate the high to the low fidelity model, a correspondence among the variables of both models was chosen. Figure 5 shows the low fidelity structure, and it consists of a total of 8 design variables: cross sections (A1, A2, A4 and A8), trusses topology (L1 and L3) and payloads (M1 and M4).

The low fidelity problem is stated as follows:

$$Minimize\ flow\ (x^{r})=CPI=w1\ Wtot+w2/(\sum Pi)+w3/(\sum Mi)$$

Subject to: $g1=1-((Mtot)min/(M1+M4))\geq 0$

$g2=1-((Ptot)min/(P1+P4))\geq 0$

$g3=1-((fn1,min)/fn1)\geq 0 \quad (22)$

$g(4\text{-}7)=1-(|\sigma\_1,2,4,8|/\sigma yield)\geq 0$

$M^{(l)}(i)\leq Mi\leq Mi^{(u)}(i)\quad i=1,4$

$L^{(l)}(k)\leq Lk\leq Lk^{(u)}(k)\quad k=1,3$

$Am^{(l)}(m)\leq Am\leq Am^{(u)}(m)\quad m=1,2,4,8$

where w1=0.003, w2=106, w3=3.5x106, (Mtot)min=2600 kg, (Ptot)min=50,000 N, fn1,min=15 s-1, M(l)1,4= 100 kg, M(u)1,4=3,000 kg, L(l)1,3=2 m, L(u)1,3=30 m, A(l)1,2,4,8=5x10-4 m2, A(u)1,2,4,8=0.01 m2. The loads P1,4 are defined to be functions of the lengths of the bays L1,3 and the payload masses M1,4. The loads are calculated using Equation (21), but L2, M2 and M3 are considered as zero.

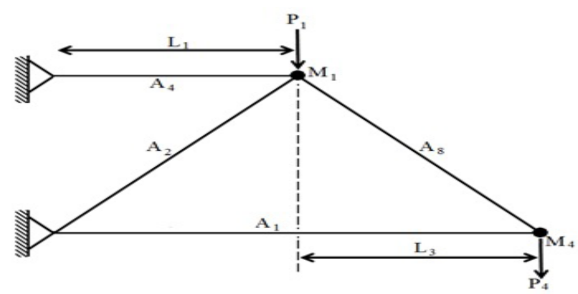

Figure 5. Physics-based low fidelity structure.

Table 5 shows the number of high fidelity function calls required by the variable fidelity model management framework for the structural optimization problem (high and low physics-based fidelity models), as a result of using the aforementioned multiplicative and additive scaling methods (first order, second order BFGS, and second order SR1).

The results show that the most noticeable difference among the methods is that the additive scaling method performs much better than the multiplicative scaling method, being the second order BFGS method the one that offered bigger savings. Furthermore, it can be observed that the multiplicative scaling second order BFGS and SR1 methods did not converge to the optimum solution. This is because optimization aborted due to the division by zero arising from the multiplicative scaling, in the objective function value of the low fidelity model. In addition, the last row shows that the use of the SQP optimizer on the high fidelity model required 453 function evaluations for convergence, which is a higher value than the required by the multiplicative and additive scaling methods.

Method	First order	Second order BFGS	Second order SR1
Multiplicative Scaling	380	-	-
Additive Scaling	382	200	289
SQP	453		

Table 5. High fidelity function evaluations for the structural optimization problem (high and low physics-based fidelity models).

4 CONCLUSIONS

The results of the problems show how models with different degree of nonlinearity and different number of design variables can be handled and integrated efficiently by the trust region model management framework, while significantly reducing the design cycle (number of high fidelity function calls) while achieving convergence.
It was observed in all test problems that the choice of the most suitable scaling methods to solve each of the problems is problem dependent, but still some points can be generalized. The following characteristics highlight in the results:

•The additive scaling method is a more robust method that performs better than the multiplicative scaling method in most cases.

•The first order methods proved to be very competitive in terms of savings, and converged to the optimum designs in all the problems.

•In some cases the multiplicative second order scaling method did not converge to the optimum. This is because optimization aborted due to the division by zero arising from the multiplicative scaling, in the objective function value of the low fidelity model.

•The use of scaling methods in the variable fidelity framework with HF and LF models that are significantly different (even in physics), could be as successful as, or more successful than using models of similar complexity or physics.

In addition, the obtained results can help to answer to the following case study scenarios, in order to be able to decide what type of scaling method is preferred, depending on the case study at hand:

•In case a LF model could be proposed, it would be desirable to build a LF model that consumed the shortest time possible, a cheap and simple model in number of variables and degree of nonlinearity is desired. In case it was considered a LF model as similar as possible to a scalar function (0 variables, 0 degree), the most appropriate methods would be the first order and second order (SR1) additive scaling methods.

•In case it is uncertain the degree of nonlinearity and the number of variables in the HF model, it would be desirable to use a robust scaling model. Some scaling methods that work well independently of the similarity between the HF and LF models are the first order and second order (SR1) additive scaling methods.

•In case there is a big, or small (or no) difference in the degree of nonlinearity between the HF and LF models. The suggested methods for a big difference would be the first order and second order (SR1) additive scaling methods. For a small difference the recommended methods would be the second order (SR1 and BFGS) additive scaling methods.

•In case there is a big, or small (or no) difference in the number of variables between the HF and LF models. The suggested methods for a big difference would be the first order and second order (SR1) additive scaling methods, and the first order multiplicative scaling method. For a small difference the recommended methods would be the second order (SR1 and BFGS) additive scaling methods.

•In case there is a big difference in the degree of nonlinearity, and a big difference in the number of variables, between the HF and LF models, the most appropriate methods would be the first order and second order (SR1) additive scaling methods.

•In case there is a small (or no) difference in the degree of nonlinearity, and a small (or no) difference in the number of variables, between the HF and LF models, the most appropriate methods would be the second order (SR1 and BFGS) additive scaling methods, and the second order (BFGS) multiplicative scaling method.

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Biographies

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